08_ARDL

QuantFit Estimator Standard Operating Procedure

SOP: Autoregressive Distributed Lag (ARDL)

Pesaran-Shin-Smith bounds testing approach

=> ARDL handles mixed I(0)/I(1) regressors and yields long-run + short-run dynamics from a single equation.

1. Purpose

ARDL embeds the dependent variable's lags and the regressors' lags in a single equation. Re-parameterising into the unrestricted error correction model (UECM) gives long-run multipliers and the speed of adjustment to equilibrium. The bounds test (Pesaran, Shin, Smith 2001) tests for cointegration without pre-determining the integration order.

2. When to use this estimator

Time-series with mixed I(0)/I(1) regressors (none I(2)).

Need both long-run elasticities and short-run dynamics.

Smaller samples (T >= 30) where Johansen is unreliable.

Single dependent variable framework.

3. Required data structure

Time series with no I(2) variables (run unit-root tests pre-estimation).

T sufficient for AIC-selected lag orders (T >= 30 typical).

Optional dummies for structural breaks identified by Zivot-Andrews.

4. Mathematical formulation

ARDL(p, q1, ?, q_k) and its UECM re-parameterisation:

Y_t = alpha + Sum_{i=1}^{p} ?_i Y_{t-i} + Sum_{j=1}^{k} Sum_{l=0}^{q_j} beta_{jl} X_{j,t-l} + epsilon_t

DeltaY_t = alpha + phi Y_{t-1} + Sum theta_j X_{j,t-1} + Sum ?_i* DeltaY_{t-i} + Sum ?_{jl} DeltaX_{j,t-l} + epsilon_t

Long-run multiplier: lambda_j = - theta_j / phi

ECT_{t-1} = Y_{t-1} - Sum lambda_j X_{j,t-1}

5. Pre-estimation diagnostics

Confirm no variable is I(2) - bounds test invalid otherwise.

Stationarity tests (ADF, PP, KPSS, DF-GLS, Ng-Perron).

Zivot-Andrews to detect structural breaks.

VIF for multicollinearity diagnosis.

6. Estimation procedure

Search ARDL(p, q1, ?, q_k) lag combinations up to maxLag.

Select the AIC-minimising combination.

Estimate the UECM by OLS and extract phi? and theta?.

Compute long-run multipliers via the delta method for SE.

Run the bounds F-test (k+1 restrictions on UECM).

Optional CUSUM / CUSUMSQ for parameter stability.

7. Output produced

8. Output interpretation

phi? < 0 and significant => stable error correction; |phi?| is the fraction of disequilibrium corrected per period.

Half-life of disequilibrium: ln(0.5) / ln(1 + phi).

lambda_j is the long-run elasticity of Y w.r.t. X_j.

Bounds F above the upper bound => cointegration; below the lower bound => no cointegration.

All Card A diagnostics should pass for inference to be valid.

9. Post-estimation diagnostics

Bounds test F and t (Pesaran-Shin-Smith critical values).

Breusch-Godfrey for residual autocorrelation.

Breusch-Pagan / White for heteroskedasticity.

Jarque-Bera for residual normality.

RESET for functional form.

CUSUM / CUSUMSQ for parameter stability.

10. Common pitfalls

If any variable is I(2), the bounds test is invalid.

Positive phi? indicates explosive dynamics - model is misspecified.

AIC vs BIC can pick different lags; document the choice.

Do not interpret long-run multipliers without confirming bounds-test cointegration.

11. Reporting checklist

Selected ARDL order [p, q1, ?, q_k] and selection criterion.

ECT coefficient with stars and half-life.

Long-run coefficient table with delta-method SE.

Short-run RECM table.

Bounds test F and t with PSS critical values.

Card A diagnostics with verdict.

CUSUM / CUSUMSQ stability comments.

12. References

Pesaran, M. H., Shin, Y., Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. Journal of Applied Econometrics.

Pesaran, M. H., Shin, Y. (1999). An Autoregressive Distributed Lag Modelling Approach to Cointegration Analysis.

Field | Meaning

ardlOrder | [p, q1, ?, q_k] selected by AIC

ect | phi? - speed of adjustment (must be negative)

ectSE / ectTStat / ectPValue | ECT inference

longRunCoefficients / longRunSE | lambda_j and SE via delta method

shortRunCoefficients | Delta-form coefficients keyed Delta(X), Delta(L(X,1)), ?

boundsTest | F-stat, t-stat, conclusion vs PSS critical values

rSquared / aic / bic | Goodness of fit

diagnostics | Card A: BG, BP, JB, RESET, CUSUM, CUSUMSQ